# A Data-Driven Gaussian Process Filter for Electrocardiogram Denoising

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## Overview

A computationally efficient data-driven Gaussian Process ($\mathcal{GP}$) filter is proposed for ECG filtering, which outperforms state-of-the-art filters in signal-to-noise ratio (SNR) improvement and results in more accurate estimates of clinically important features of the ECG, such as the QT-interval.

## $\mathcal{GP}$-based ECG modeling

- **Model:** the noisy measurements $x(t)$ are modeled as a mixture of the clean ECG $s(t)$ and additive uncorrelated noise $n(t)$, at time instants $t$ for which measurements are available:

$$x(t) = s(t) + n(t), \quad t \in \{t_1 \dots t_N\}$$

- **Assumption:** the noise $n(t)$ and the ECG $s(t)$ are Gaussian processes:

$$s(t) \sim \mathcal{GP}\left(\mu_s(t), \kappa_s\left(t, t'\right)\right)$$
$$n(t) \sim \mathcal{GP}\left(\mu_n(t), \kappa_n\left(t, t'\right)\right)$$ , typically with $\mu_n(t) = 0$ and $\kappa_n\left(t, t'\right) = \sigma_n^2 \delta\left(t, t'\right)$

- **Note:** A $\mathcal{GP}$ is fully described by its mean and covariance matrix. Therefore, the ECG data-model is completely specified by the choice of the mean and covariance matrices of the $\mathcal{GP}$.
- Denoting $\boldsymbol{\mu_s} = [\mu_s(t)]_t$ as the signal mean, $\boldsymbol{K_s} = [\kappa_s\left(t, t'\right)]_{t,t'}$ as the signal covariance matrix (kernel), and $\boldsymbol{K_x}$ as the covariance matrix of the measurements, the optimal maximum a posteriori (MAP) estimator (filter) of the ECG is:

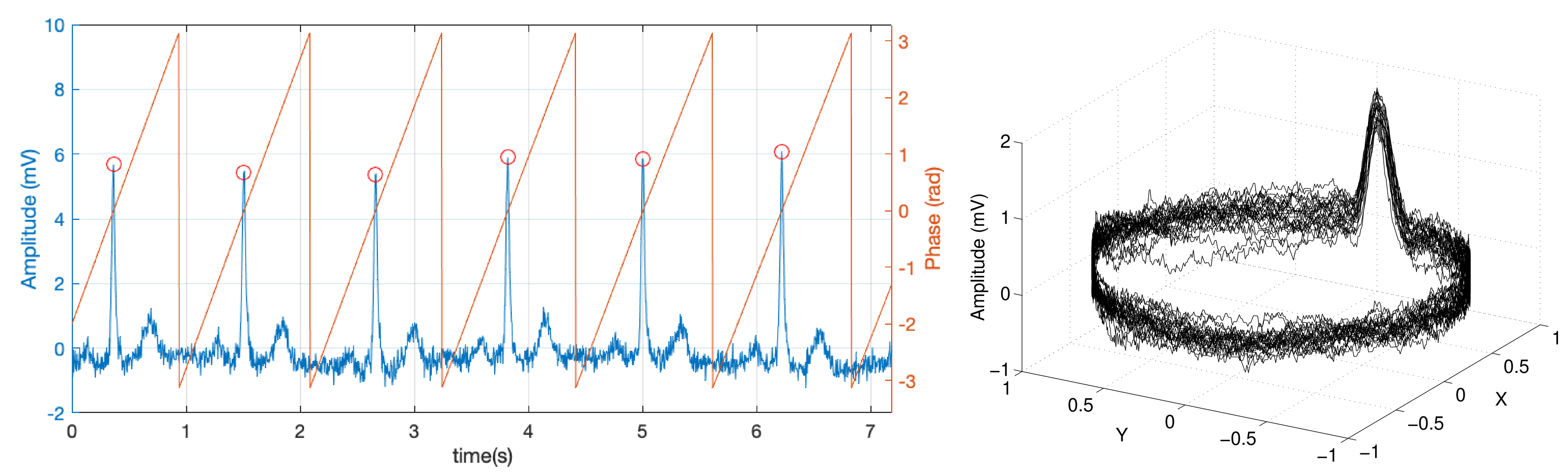
$$\widehat{\boldsymbol{s}} = \boldsymbol{K_s}\boldsymbol{K_x}^{-1}(\boldsymbol{x} - \boldsymbol{\mu}_s) + \boldsymbol{\mu}_s$$

## $\mathcal{GP}$-based ECG modeling challenges

- The choice of the covariance matrix $\boldsymbol{K_s}$ is not evident (*analytical kernels proposed in the literature [2], [3], require optimizing over a large number of hyper-parameters*).
- The inversion of $\boldsymbol{K_x}$ required by the filter restricts the use of the model to short ECG signals (the complexity of matrix inversion is $\mathcal{O}(N^3)$).

## A data-driven $\mathcal{GP}$-based filter

- **The proposed $\mathcal{GP}$ scheme:** The kernel is built based on the sample covariance matrix corresponding to the ECG in the *phase domain* [6], via a transformation $\boldsymbol{\Theta} \in \mathbb{R}^{\mathcal{T} \times N}$, which maps each ECG beat to $\mathcal{T}$ bins in the phase domain (regardless of the heart rate).




(a) ECG and ECG phase (b) Phase wrapped ECG

Figure 1. A typical ECG and the ECG phase that maps each ECG cycle to $[-\pi, \pi]$ (cf. [6])

**Algorithm:** 1) detect the R-peaks; 2) transform the ECG to the phase domain; 3) calculate the $\mathcal{GP}$ mean vector and covariance matrix; 4) estimate the noise covariance; 5) apply the filter.

## Data-driven $\mathcal{GP}$ priors selections

- A transformation matrix $\boldsymbol{\Theta}$ maps the ECG to the mean "*binned-heartbeat*" $\boldsymbol{x}_{\text{ph}}$ in the phase domain:

$$\boldsymbol{x}_{\text{ph}} = \boldsymbol{\Theta}\boldsymbol{x} \qquad \left[\mathsf{S}_{s_{\text{ph}}} = \boldsymbol{\Theta}\boldsymbol{K}_s\boldsymbol{\Theta}^T\right].$$

- The transformation is chosen such that $\boldsymbol{\Theta}\boldsymbol{\Theta}^T = \boldsymbol{I}_{\mathcal{T}}$.
- In phase domain, the sample covariance matrices are linked by

$$\mathsf{S}_{s_{\text{ph}}} = \mathsf{S}_{x_{\text{ph}}} - \tilde{\sigma}_n^2 \boldsymbol{I}_{\mathcal{T}}.$$

- The Kernel choice is $\boldsymbol{K}_s = \boldsymbol{\Theta}^T \mathsf{S}_{s_{\text{ph}}} \boldsymbol{\Theta}$ and the mean choice is $\boldsymbol{\mu}_s = \boldsymbol{\Theta}^T \boldsymbol{\mu}_{\text{ph}}$, where $\boldsymbol{\mu}_{\text{ph}}$ is the average ECG beat in the phase domain (both obtained in a data-driven non-parametric approach).
- The inversion is performed via the matrix inversion lemma:

$$\boldsymbol{K}_x^{-1} = \sigma_n^{-2}(\boldsymbol{I}_N - \boldsymbol{\Theta}^T\boldsymbol{\Theta}) + \boldsymbol{\Theta}^T \mathsf{S}_{x_{\text{ph}}}^{-1} \boldsymbol{\Theta}.$$

- The filter is data-driven and requires the inversion of a $\mathcal{T} \times \mathcal{T}$ matrix (typically $\mathcal{T} \ll N$):

$$\widehat{\boldsymbol{s}} = \boldsymbol{\Theta}^T(\boldsymbol{I}_{\mathcal{T}} - \tilde{\sigma}_n^2 \mathsf{S}_{x_{\text{ph}}}^{-1})\boldsymbol{\Theta}(\boldsymbol{x} - \boldsymbol{\mu}_s) + \boldsymbol{\mu}_s.$$

## Implementation

- In the simplified scheme, only the sample variances are taken into account and the filter becomes:

$$\widehat{\boldsymbol{s}} = \boldsymbol{\Theta}^T \mathbf{P} \boldsymbol{\Theta}(\boldsymbol{x} - \boldsymbol{\mu}_s) + \boldsymbol{\mu}_s, \quad \mathbf{P} = \text{diag}\left[\dots\ 1 - \rho_i\ \dots\right], \quad \rho_i = \frac{\tilde{\sigma}_n^2}{\sigma^2_{x_{\text{ph}_i}}} = \frac{\tilde{\sigma}_n^2}{\sigma^2_{s_{\text{ph}_i}} + \tilde{\sigma}_n^2}.$$

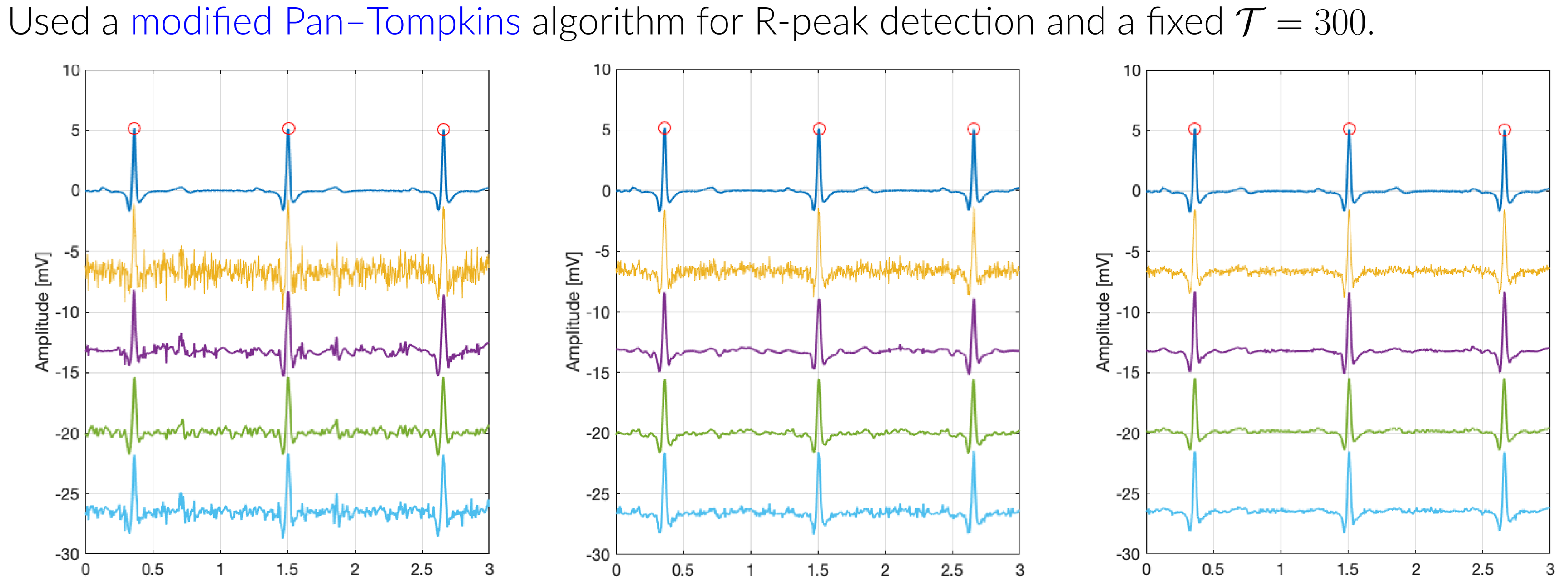
- In accordance with Bayesian filtering terminology, we refer to the above as $\mathcal{GP}$ posterior-based, and the corner case of extremely noisy ECG ($\tilde{\sigma}_n^2 \to \infty$) as $\mathcal{GP}$ prior-based.

## Advantages

- The model's kernel is calculated in a data-driven manner (no *ad hoc* parametric kernels are required).
- As compared with conventional $\mathcal{GP}$, the computational cost is significantly reduced.
- The model requires only *two* hyper-parameters: 1) the number of phase-domain bins $\mathcal{T}$ (which can be fixed); 2) the noise covariance (which is estimated via maximum evidence).

## Results

- **Dataset:** The PhysioNet QT Database [5]. **Benchmark:** a wavelet denoiser, which outperformed other non-model-based filtering schemes in previous research [4]
- Used a modified Pan–Tompkins algorithm for R-peak detection and a fixed $\mathcal{T} = 300$.




(a) SNR = 0dB (b) SNR = 6dB (c) SNR = 12dB

Figure 2. An ECG from the PhysioNet QT Database at different SNRs denoised by the proposed filters and a wavelet denoiser [4]. From top to bottom: the original ECG, noisy, wavelet, $\mathcal{GP}$ prior-based and $\mathcal{GP}$ posterior-based

## SNR improvement results

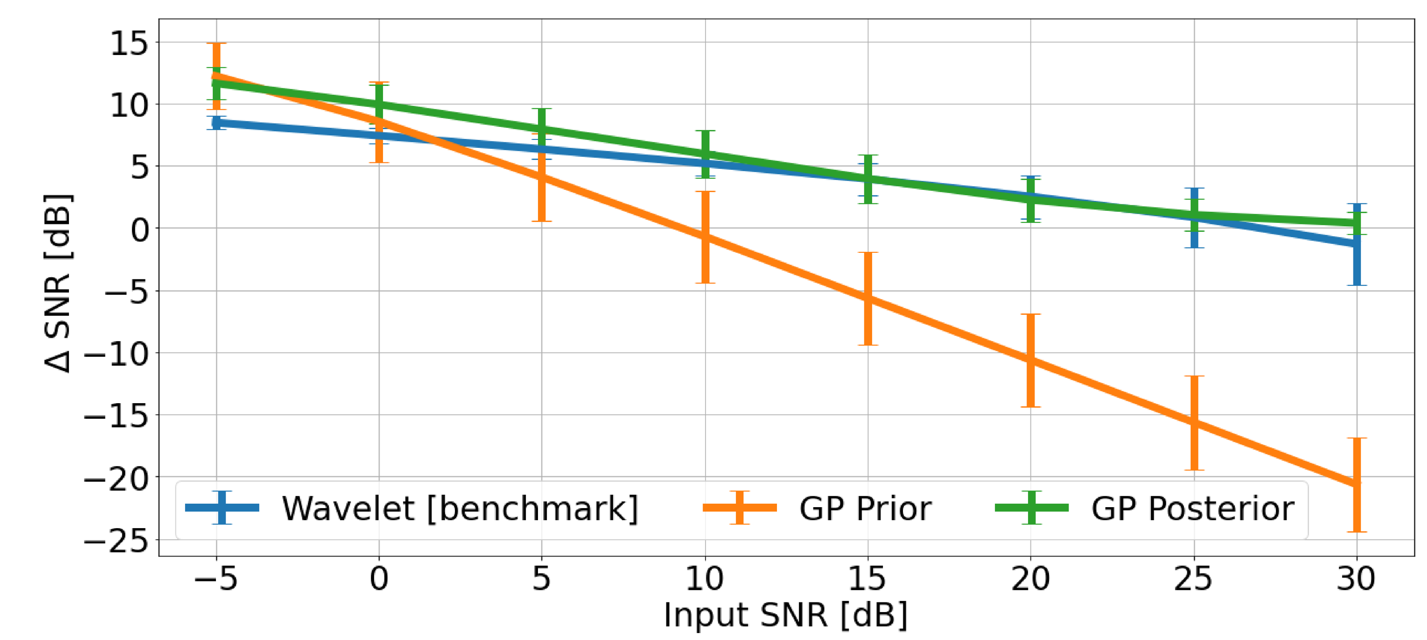



Figure 3. SNR improvement using the proposed $\mathcal{GP}$ filter and the benchmark wavelet denoiser [4]. The curves correspond to the average SNR improvement across all samples of the PhysioNet QT Database, in leads I and II (whenever available) and with 5 repetitions using different random noise instances per record.

## QT estimation performance

- The proposed filter also preserves clinically important features of the ECG, such as the QT-interval.
- We use the algorithms detailed in [1] to compare the QT-intervals estimated from clean ECG and the ones estimated from the denoised ECG using the proposed and benchmark wavelet denoisers.

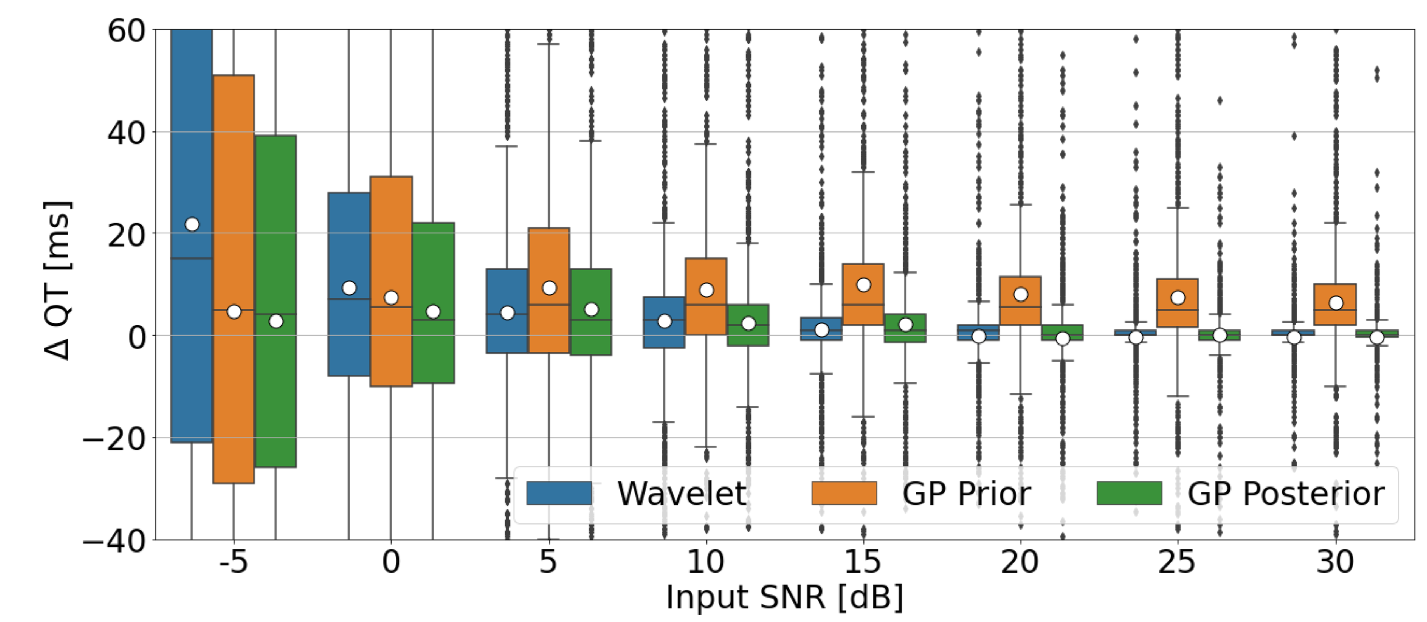



Figure 4. The distribution of the QT-interval estimation differences between the outputs of the proposed and benchmark algorithms minus the QT-interval estimated from the clean ECG (at different SNR). The $\mathcal{GP}$ posterior-based filter outperforms the prior-based and the state-of-the-art wavelet denoiser, in terms of estimation error bias and variance.

## Conclusion

- Compared with the state-of-the-art non-model-based ECG denoisers, the proposed data-driven $\mathcal{GP}$ posterior-based filter has superior performance in terms of SNR improvement and QT-interval parameter estimation accuracy.

## References

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